PROPERTIES OF |X|

(i) $\sqrt{x^2} = |x| \rightarrow |x|^2 = x^2 \ \forall\ x \in R$ (ii) $||x|| = |x| = |-x| \ \forall\ x \in C$

(iii) $|xy| = |x| \cdot |y| \ \forall\ x, y \in C$. The property extends to $|xyzw| = |x||y||z||w|$ & $|x^n| = |x|^n$

(iv) If $|x| = 0$ then $x = 0$ (v) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$, where $x, y \in C$ & $|y| \neq 0$

(vi) Triangular Inequality $||x| - |y|| \leq |x \pm y| \leq |x| + |y| \ \forall\ x, y \in C$

(vii) If $x, y \in R$ and $|x + y| = |x| + |y|$ then both x and y are of same sign, i.e. $xy \geq 0$.

(viii) If $x, y \in R$ and $|x - y| = |x| + |-y|$ then both x and - y are of same sign, i.e. $xy \leq 0$.

(ix) $|x| = x \rightarrow x \geq 0$, the property can be extended to function as well. $|f(x)| = f(x) \rightarrow f(x) \geq 0$.

(x) $|x| + x = 0$ or $|x| = -x \rightarrow x \leq 0$. Similarly, $|f(x)| = -f(x) \rightarrow f(x) \leq 0$.

(xi) If $|x| + |y| = 0 \rightarrow x = 0$ and $y = 0\ \forall\ x, y \in C$

(x) $|x| \geq 0\ \forall\ x \in R$

(xi) $x^2 + y^2 = 0 \rightarrow x = 0, y = 0$ only in real numbers not in complex numbers.

(xii) $1 \leq |\sin(x)| + |\cos(x)| \leq \sqrt{2}\ \forall\ x \in R$

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1. If $|x - 2| = 5$ & $|y| = 4$ then select the correct options

   (A) $x = -1$ or 7 (B) $y = \pm 4$ (C) $x + y \in \{-5, 3, 11\}$

   (D) Sum of all the different possible values of $|x| + |y|$ is 16.

2. If $|xy| = 6$ & $x, y \in I$ then minimum value of $|x|^{|y|} + y + x$, is

   (A) 4 (B) - 5 (C) - 6 (D) 1

3. The number of real roots of the equation $|x|^2 - 3|x| + 2 = 0$, is

   (A) 1 (B) 2 (C) 3 (D) 4

4. Find the real roots of the equation $|x|^2 - 2|x| - 3 = 0$.

5. Solve the equation $|2x - 3| = 8$.

6. Find the number of solutions of equation $|xy| = |y|$, hence plot the solution curve on Cartesian Co-ordinate system.

7. Solve the following Equations

   (i) $|x + 2| = 2(3 - x)$ (ii) $|3x - 2| + x = 11$

   (iii) $||x - 1| + 2| = 1$ (iv) $|x-2|^2 + |x-2| - 2 = 0$

   (v) $x^2 - |x| - 2 = 0$

8. The number of real roots of the equation $x^2 - 4x - 3|x - 2| + 6 = 0$ is/are?

(A) 0 (B) 2 (C) 3 (D) 4

9. $|x + 1| + |x - 1| + |x + 2| + |x - 2| = 0$ then x has

(A) The values {1, –1, 2, –1} (B) the values $\left\{\frac{1}{2}, -\frac{1}{2}, \frac{3}{2}, -\frac{3}{3}\right\}$

(C) No solution (D) none of these

10. If $\frac{1}{|x|} + \frac{1}{x} = 1$ then the value of x is/are

(A) 1 (B) 2 (C) 3 (D) None of these

11. If ordered pairs (x, y) satisfies $|3x - 1| = 2$ and $|y - 1| = 1$ then the correct option(s) is/are

(A) Only 3 pairs (x, y) are possible

(B) 4 pairs (x, y) are possible

(C) The maximum area formed by line joining pairs = 8/3 sq. unit

(D) No of possible matrices $\begin{bmatrix} -x & y \\ -y & x \end{bmatrix}$ are 4.

12. The minimum value of $|x - 1| + |x - 3| + |x - 4|$ is

(A) 5 (B) 4 (C) 3 (D) None of these

13. The number of integral values of x satisfying $|(x - 1)(x - 7)| > x^2 - 8x + 7$, are

(A) 5 (B) 4 (C) 3 (D) None of these

14. Solve the following equations

(i) $|x - 1| + |x - 2| = 2$ (ii) $|x - 1| + |x - 2| = 3$

(iii) $|x| - |x - 1| = 1$ (iv) $|2x - 1| + |x| = 5$

15. Solve the equation $|2x - 1| = |x + 2|$

16. Find the maximum and minimum value of $|x + y|$ if $x \in [-1, 2]$ & $y \in [-1, 1]$

17. Find the minimum value of the expression $F(x) = |x - 1| + |x - 2| + |x - 3| + \ldots\ldots + |x - 50|$

Where $x \in R$

18. (i) Solve :

(a) $|x - 1| < 2$ (b) $|x - 3| \geq 3$ (c) $|x - 5| < -6$ (d) $|x - 3| > -5$

(e) $2 < |x - 1| \leq 3$ (f) $2 < |3x - 2| < 5$

(ii) $|x-1| \leq 4 \leq |x+1|$, then

(A) $x \in [3, 5]$ (B) $x \in [-3, 5]$ (C) $x \in [-1, 1]$ (D) $x \in (-\infty, -3) \cup (5, \infty)$

19. $3 < |x - 1| < 4$ and $-1 < |x - 2| < 6$, then $x \in$

(A) x = {1, 2, 3} (B) $x = \left\{\frac{3}{2}, \frac{5}{2}\right\}$ (C) Φ (D) none of these

20. $||x-1|-2| \leq 5$, then $x \in$

(A) $x \in [-3,7]$ (B) $x \in [-8,8]$ (C) $x \in [-6,8]$ (D) none of these

21. $\left||x-4|-3\right| \geq 6$, then $x \in$

(A) $x \in (-\infty,-5] \cup [13,\infty)$ (B) $x \in (-\infty,-3] \cup [9,\infty)$

(C) x has no solution (D) only x = {-5} or {13} or {20}

22. Solve the following inequalities

(i) $|x^2 - 2x + 2| \geq 4$ (ii) $\left|2-\sqrt{2-x}\right| \leq 0$

(iii) $3 < |x^2 - x| \leq 4$ (iv) $|x|^3 - 2x^2 - 4|x| + 3 < 0$

(v) $|x^2 + 6x + 7| = |x^2 + 4x + 4| + |2x + 3|$ (vi) $|x^2 - 7x + 12| > x^2 - 7x + 12$

(vii) $|3x - 5| - |2x + 3| > 0$ (viii) $|x^2 - 2x - 3| < 3x - 3$

(ix) $|x^2 - 5x| > |x|^2 - 5|x|$

ANSWER KEY

1. B 2. C 3. D 4. $x = \pm 3$

5. $x = 11/2$ or $-5/2$.

7. (i) $x = 4/3$ (ii) $x = -\frac{9}{2}$ and $\frac{13}{4}$. (iii) no solution (iv) x = 3 and 1

(v) $x = \pm 2$

8. D 15. x = 3 and – 1/ 3